A response to the papers by Hehl, and Hehl and Obukhov

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Abstract

A detailed scientific rebuttal is given to the criticisms by Hehl and Obukhov of Einstein Cartan Evans (ECE) unified field theory that were published in two papers in *Foundations of Physics* 38(1). It is shown that both papers contain basic errors and misunderstandings. In addition, it is pointed out that citations of other criticisms of ECE theory in the two papers are made without citing existing rebuttals. Some of these rebuttals have been available for over a decade in the literature, and are either referenced in this paper or reproduced in appendices for the record. This new rebuttal is structured in the same way as the papers of Hehl and Obukhov for ease of comparison. More than 80 detailed points of error, misunderstanding or confused thought are discussed – this represents a small sample of the errors that exist. They are chosen to illustrate that these critical papers demonstrate a clear misunderstanding of ECE theory, often at a very basic level.

Keywords: Einstein Cartan Evans unified field theory.

Introduction

This refutation addresses the following two papers that were published in *Foundations of Physics* 38(1)**:** Hehl, F. W. (2007), *An Assessment of Evans' Unified Field Theory I*, 7-37; and Hehl, F. W. and Obukhov, Y. N. (2007), *An Assessment of Evans' Unified Field Theory II,* 38-46*.*

Einstein Cartan Evans (ECE) unified field theory has recently been completed (the first fifteen foundational papers have been formally accepted through the standard anonymous peer review process of the journal *Foundations of Physics Letters*, between 2003 and 2005).

The number of criticisms that have been published or reported are relatively few (and restricted to a handful of scientists worldwide). They have been responded to, and misunderstandings and errors have been corrected, in the accepted scientific manner.

ECE theory is fundamentally based on long accepted and widely used Cartan geometry [13]. Established in 1922, Cartan Geometry is taught in courses in university departments around the world. In the papers and books on the aias.us website, ECE theory has been rigorously tested for self-consistency and against experimental data in a systematic way in the various fields of physics, and shown to reduce in appropriate limits to all of the well-known laws of physics.

ECE theory unifies classical and quantum mechanics consistently and coherently for the first time, making concepts such as uncertainty obsolete. It reduces to all of the well-known equations of quantum mechanics against which it has so far been tested. It has numerous experimental advantages over the Standard Model, reproducing the results of many experiments that the Standard Model cannot explain. It makes notions such as Dark Matter unnecessary by showing that there is no missing energy in the universe.

It will be shown in this paper that the current criticisms demonstrate a fundamental misunderstanding of the theory, and as a consequence contain many errors and misconceptions. Most of these errors and misunderstandings have already been corrected rigorously in previous rebuttals, but Hehl and Obukhov fail to cite these rebuttals. This paper, therefore, assesses in detail the remarks by Hehl and Obukhov, and highlights the main points of their scientific inaccuracies. It reviews their misunderstandings and errors (and those of other authors that they cite), and shows the necessary corrections that need to be made to acquire a correct understanding of ECE theory. It contains numerous rebuttal statements, and technical appendices outlining some of the key rebuttals in more mathematical detail, as and when considered appropriate.

The validity, relative simplicity, and great strength of ECE theory is the result of the fact that it is based directly on Cartan geometry. Some misleading remarks by Hehl and Obukhov concerning the basic ECE hypothesis are also corrected.

[1.1] Abstract and Introduction

In the abstract of the first paper by Hehl, it is claimed that ECE theory is strictly classical, whereas it has been shown to be fully quantized and has been applied to quantum field theory (unifying consistently, and for the first time, classical and quantum mechanics). That is, ECE theory succeeds in unifying general relativity and quantum mechanics where the Standard Model is well known to fail. Hehl repeats elementary errors by Bruhn in a series of citations of unrefereed website documents, while existing rebuttals are not cited. For this reason, some already published rebuttals are summarized again in appendices attached to this paper.

Hehl claims, for example, that there is no anti-symmetric metric, whereas this object is clearly defined in ECE theory as the wedge product of tetrads. This is a definition, so Hehl effectively states that a wedge product of tetrads cannot be made, this is reductio ad absurdum. The symmetric metric is well known to be the scalar product of tetrads. Hehl claims further that the wedge product of two tetrads is somehow inconsistent with Cartan geometry [13].

A wedge product is well defined. The equation

$$D \wedge T^a_b = 0 \tag{1}$$

follows from the second Bianchi identity:

$$D \wedge R^a_b = 0\,, \tag{2}$$

where R^a_b and T^a_b are well defined [1-12]. Hehl does not appear to be aware of this fact.

Both Hehl and Rodrigues claim that the tetrad postulate is somehow not applicable, whereas it is well known and long accepted that the tetrad postulate is the fundamental requirement that a vector field be independent of its components and basis elements. The tetrad postulate always holds for any application in any area of science and natural philosophy.

Hehl then makes some obscure remarks about the constant of proportionality $A^{(0)}$ in the fundamental ECE hypothesis:

$$A^a_\mu = A^{(0)} q^a_\mu \ , \tag{3}$$

where A^a_μ is the electromagnetic potential form and q^a_μ is the Cartan tetrad form. If c is the velocity of light, then $cA^{(0)}$ has S.I. units of volts:

$$A^{(0)} = JsC^{-1}m^{-1} \ , \ cA^{(0)} = JC^{-1} = \text{ Volt} \, . \tag{4}$$

Hehl also makes some confused remarks about the transformation properties of A^a_μ. Its transformation properties are those of the tetrad [1-13], a vector valued one-form, and are discussed in comprehensive detail on aias.us. They are

$$A^{a'}_{\mu'} = \Lambda^a_{a'} \frac{\partial x^\mu}{\partial x^{\mu'}} A^a_u \ , \tag{5}$$

where $A^{a'}_a$ is the Lorentz transform, and where $\frac{\partial x^\mu}{\partial x^{\mu'}}$ denotes general coordinate transformation. This is entirely standard and well known, since it is a basic part of Cartan geometry.

As an example of obfuscation, Hehl accepts that the ECE wave equation is technically correct but proceeds to describe it as "redundant". This is nonsense because it would imply that the tetrad postulate is "redundant". It is claimed that the paper by Hehl and Obukhov [15] constructs a Lagrangian in an original manner. Lagrangians for ECE theory are already available [1-12]. Hehl points out that ECE theory is entirely original to the present author, and claims that Cartan did not suggest his geometry to Einstein. This claim concerning Cartan may or may not be correct. Hehl then makes some remarks about others who have apparently used Cartan geometry, but all of this is largely irrelevant to the intended discourse on ECE theory.

[2] Geometry: Riemann-Cartan Geometry of Spacetime

In Section 2 of the first paper [14], standard textbook type material is introduced that is irrelevant to ECE theory [1-12]. The latter is based on the two standard Cartan structure equations and the two standard Bianchi identities. These have been rigorously tested [1-12] in many ways for self-consistency, and have been reduced [1-12] to their equivalents in Riemann notation in order to demonstrate self-consistency. ECE theory has been rigorously tested against experimental data in many ways (aias.us), successfully reproducing the results of many experiments that the Standard Model fails to explain for reasons that become clear within the ECE theory framework. Many problems of physics that have existed for a long time are resolved, for example, the velocity curves of stars in galaxies. Furthermore, new phenomena are being predicted, like resonance effects due to spin connection resonance, which is a technologically important new insight provided by ECE theory.

Heil cites Bruhn who claims that the tetrad postulate is somehow “incorrect”, so according to Bruhn, Cartan is incorrect. This inconsistency is compounded by his citation of Rodrigues, who also claims that the tetrad postulate is “incorrect”. However, Hehl cites none of the relevant rebuttals. He also cites Lakhtakia who claims, on the other hand, that Cartan geometry is correct. We need no knowledge of mathematics to see that there is internal inconsistency in these citations, and in the confused train of thought that is scattered throughout these papers.

Hehl makes the claim that the Cartan torsion is unrelated to spin. This is untrue. The first Cartan structure equation states in the notation [13] of Cartan geometry that

$$T^a = d \wedge q^a + \omega^a_b \wedge q^b \quad , \tag{6}$$

where T^a is the Cartan torsion form, $d \wedge$ is the exterior derivative, realized by wedge product, and ω^a_b is the spin connection form. In tensor notation Eq. (6) becomes

$$T^a_{\mu\nu} = \partial_\mu q^a_\nu - \partial_\nu q^a_\mu + \omega^a_{\mu b} q^b_\nu - \omega^a_{\nu b} q^b_\mu \quad , \tag{7}$$

and clearly has the anti-symmetry needed for angular momentum or torque. These quantities involve spin and this is meticulously defined in ECE theory [1-12], in many places. Hehl obviously misunderstands the original points in ECE theory (see the appendices to this paper), because spin torsion and orbital torsion have been clearly defined in ECE theory. Hehl cites other work concerning angular momentum, but that is not ECE theory. He states, in a footnote, that “we are not told what sort of spin we have to think of”. Spin torsion and orbital torsion are clearly defined in ECE theory [1-12], and so are all dynamical quantities, in an original way. In a footnote to Section 2.3, Hehl again agrees that Cartan geometry as used by Carroll [13] and the present author [1-12] is correct. Yet he then cites Bruhn and Rodrigues (again), who claim that it is incorrect. So Hehl et al. are self-contradictory. In Section 2.3, there is (again) a long and irrelevant textbook-type reiteration that is not relevant to ECE theory.

In Section 2.4, Hehl confuses the ECE Lemma [1-12] with what he calls the “Ricci Identity”. In this confused state he then asserts that there is an error in the ECE Lemma. The ECE Lemma is obtained from the tetrad postulate in a simple and self-consistent manner (for example, see Appendix J of Ref. [1]). Hehl states that Cartan is correct, Bruhn and Rodrigues state that Cartan is incorrect – so a central part of Hehl's thesis is actually based on the validity of Cartan geometry itself. Needless to say, the entire community of twenty-first century mathematicians [13] accept its validity, and regularly teach Cartan geometry (the basis of ECE theory) in university departments.

[3] Electromagnetism: Evans' Ansatz for Extended Electromagnetism

In Section 3.1, Hehl states once more that ECE theory is technically correct, i.e., "Up to now everything is conventional". He states thereafter that "Evans has a highly unconventional ad hoc ansatz" (sic). This obscure grammar presumably means that the ECE hypothesis is original (but in an unconventional way?). Since every hypothesis must be original, this statement is a tautology.

In a footnote to Section 3.1, Hehl contradicts his previous statements that Cartan geometry is correct, now claiming that it is incorrect, and that Cartan has "committed a mistake". The mathematical truth (see the appendices) is that the two Cartan structure relations and the two Bianchi identities are rigorously self-consistent within their well-defined terms of definition, as Hehl himself admits in other parts of the same paper. What are we to make of this porridge of self-contradictions?

Hehl compounds his confusion by further subjective statements that have no place in natural philosophy. He claims that $A^{(0)}$ must be a universal constant on the basis of units analysis. If so, $cA^{(0)}$, being a voltage, must be universal. The truth is that $A^{(0)}$ is the scalar magnitude of a vector potential and, as shown in the appendices, the minimum $cA^{(0)}$ is a universal constant. The four potential is what Feynman described as a universal influence in gauge theory, but this does not mean that its magnitude is a universal constant. This is a basic confusion of concepts by Hehl, who states that "this doesn't smell particularly universal". In the appendices, it is shown that the minimum $cA^{(0)}$ is $\frac{mc^2}{e}$, where mc^2 is the rest energy of the photon and e is the charge on the proton. This is a universal constant. Hehl makes some apparently erroneous attempts at defining the units of $cA^{(0)}$; they are volts.

Hehl's confusion is further compounded by statements about the transformation properties of the A_μ^a form. It is well known that a vector valued one-form transforms according to Eq. (5). Again, this is explained in detail in ECE theory [1-12], detail which Hehl never cites. Hehl states arbitrarily that A_μ^a must be identified with a Maxwellian potential. This is an obscure claim. If it is based on Lorentz covariance it is untrue, because ECE is generally covariant, not Lorentz covariant as in what Hehl wrongly describes as Maxwell's theory. Presumably he is referring to the Maxwell Heaviside theory. The correct way to reduce A_μ^a to a Maxwell Heaviside potential is to use complex circular indices for a and Cartesian indices for μ. Hehl does not seem to understand this, as his statements reveal. The nature of A_μ^a has also been explained in detail in ECE theory.

Hehl states that he "would like to kill the a index". The a index is that of circular polarization, and indicates the fundamental complex circular basis. This is just as fundamental as the Cartesian basis or spherical polar basis, for example.

Hehl obscurely refers to "O(3) covariance". Presumably he is referring to the B Cyclic Theorem of the mid-nineties:

$$\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = iB^{(0)}\boldsymbol{B}^{(3)^*}$$

et cyclicum, (8)

which has O(3) symmetry. Bruhn's incorrect assertions concerning this theorem are corrected in the appendices. The electromagnetic field form in ECE theory is proportional to the Cartan torsion form, a vector valued two-form whose transformation properties are determined by well-known and well-accepted geometry [1-13]. Hehl again cites Bruhn who falsely claims that the equations (8) are somehow "not covariant". The truth is that the equations (8) are the equations of the frame itself, and the frame itself is well known to be covariant (see the appendices).

In his Eq. (31), Hehl manages to see the correctness of

$$F^a_{\mu\nu} = A^{(0)} T^a_{\mu\nu} \ , \tag{9}$$

where $F^a_{\mu\nu}$ is the electromagnetic field form of ECE theory. This is a symmetry conserving equation as follows:

$$F^a_{\mu\nu} = -F^a_{\nu\mu} \ , \tag{10}$$

$$T^a_{\mu\nu} = -T^a_{\nu\mu} \ . \tag{11}$$

Hehl reiterates well-known textbook material concerning the form notation of the Maxwell Heaviside field theory, but exhibits a lack of understanding of that textbook material in his claim that the homogeneous and inhomogeneous equations are independent. It is well known that they are related in the vacuum by the transform:

$$\boldsymbol{E} = -ic\boldsymbol{B} \ . \tag{12}$$

The Maxwell Heaviside equations that he cites (and confuses with the Maxwell quaternion equations) are obtained in the limit when the spin connection goes to zero. For example, the homogeneous field equation of ECE theory is

$$d \wedge F^a = \mu_0 j^a \ , \tag{13}$$

where μ_0 is the S.I. permeability in vacuo, and where j^a is the ECE homogeneous current.

When there is no interaction between gravitation and electromagnetism:

$$j^a = 0 \tag{14}$$

and

$$d \wedge F^a = 0 \ , \tag{15}$$

which for each polarization index a is the homogeneous equation of Maxwell Heaviside field theory. Q.E.D. The only thing left for Hehl to question is circular polarization itself.

In his Section 3.2, Hehl falsely claims that the Lorentz force equation has not been obtained in ECE theory. In truth, the Lorentz force equation has been obtained [1-12] from the transformation properties of the field form F (see the appendices). Hehl correctly cites the homogeneous ECE equation in his Eq. (37) (using his own obscure notation) but, not satisfied with this, he claims that the homogeneous ECE equation is an “analog” of what he calls Maxwell theory, presumably Maxwell Heaviside theory. He misses the fact that ECE is much more general than Maxwell Heaviside theory. In the paragraph following his Eq. (39), Hehl tries to find ways of stating that a correct equation is incorrect, but on the other hand correct. He then states that his Eq. (29) is the “real” ECE homogeneous field equation; however, the truth is that Eq. (13) of this rebuttal is the ECE homogeneous field equation [1-12]. Hehl then states that the homogeneous current of ECE theory is “strange”. This is unscientific subjectivity. It is simply a quantity that has no counterpart in any other theory.

The objective and geometrical truth is that the homogeneous current of ECE theory is defined by

$$j^a := \frac{A^{(0)}}{\mu_0}\left(R^a_b \wedge q^b - \omega^a_b \wedge T^b\right) \ , \tag{16}$$

where R^a_b is the Cartan curvature. The current (16) is obtained straightforwardly from the first Bianchi identity:

$$d \wedge T^a + \omega^a_b \wedge T^b := R^a_b \wedge q^b \ , \tag{17}$$

by rearranging terms as follows:

$$d \wedge T^a := R^a_b \wedge q^b - \omega^a_b \wedge T^b \ . \tag{18}$$

Now multiply both sides by $cA^{(0)}$ (the primordial voltage). By using the definition (16), we obtain Eq. (13). What is strange about this simple algebra? Contrary to Hehl’s claims, the transformation properties of both sides of Eq. (13) are the same; they are those of a vector valued three-form [1-13]. Hehl then criticizes the ECE Lemma and wave equation. The ECE wave equation and Lemma are obtained straightforwardly from the tetrad postulate as in Ref. [1], Appendix J.

Hehl’s claims that the homogeneous field equation (13) somehow represents an “additional assumption” are nonsensical. Additional to what? It is obtained directly from the Bianchi identity. Then we are told that the homogeneous field equation is not Lorentz covariant. We all know that the homogeneous equation is not Lorentz covariant, it is generally covariant. In his Eq. (46), Hehl appears to revert to agreement once more, and cites the present author’s inhomogeneous equation correctly. He correctly cites the inhomogeneous ECE equation in his Eq. (48), albeit in a different notation. He then asserts, incorrectly, that this has been derived from a Hodge dual transformation of $d \wedge F^a$, whereas he himself has just agreed that it is derived from a Hodge dual transform of F^a itself, not of $d \wedge F^a$.

He then makes the tautological deduction that the inhomogeneous current cannot be derived from a Hodge dual of $d \wedge F^a$. In fact, he himself has just shown that the inhomogeneous current has been derived from a Hodge dual of F^a. The truth is that, as Hehl himself states in his own equation (46), the same Hodge transformation is correctly applied on both sides of the equation to the anti-symmetric tensors present on both sides of the equation. He describes this as a “recipe”, but it is straightforward algebra.

Following his Eq. (48), we are given a whole series of consequentially erroneous comments.

In his Section 3.5, it is again falsely stated that the Lorentz force equation has not been derived in ECE theory. The truth is [1-12] that it has been derived from the transformation properties of the F^a form (see the appendices). At this stage in this immensely long document by Hehl [14], sequentially erroneous comments are compounded, comments that merely compound an initially false claim. We are then told that the homogeneous and inhomogeneous currents of ECE theory are mysteriously "non-covariant". The fact is that both are vector valued three-forms which transform as such under the usual rules of Cartan geometry [1-13]. The correct way to look at the general covariance is to transform both sides of the original equation (in indexless notation):

$$D \wedge F = R \wedge A \quad , \tag{19}$$

which within $A^{(0)}$ is the first Bianchi identity:

$$D \wedge T = R \wedge q \quad . \tag{20}$$

This transforms to

$$D' \wedge T' = R' \wedge q' \quad , \tag{21}$$

then rearrange terms:

$$d' \wedge T' = R' \wedge q' - \omega' \wedge T' \tag{22}$$

and

$$d' \wedge F' = R' \wedge A' - \omega' \wedge F' \quad . \tag{23}$$

For all quantities, use the rules for general coordinate transformation given by Carroll. It is in any case obvious that the first Bianchi identity is generally covariant, no matter how it is rearranged, provided the coordinate transform is carried out correctly, as above.

[4] Gravitation: Evans Adopted Einstein-Cartan Theory of Gravity

In Section 4, Hehl appears to revert to agreement once more but decides to reinterpret the author's original ECE theory in terms of his own theory, claiming incorrectly that ECE theory is after all somehow "not original". If it were not original, Hehl would have had no motive to write this long document. Several claims are made and falsely attributed to the present author.

The truth is that the dynamics of ECE theory are exemplified as follows:

$$d \wedge T^a = j^a \ , \ d \wedge \tilde{T}^a = J^a \quad , \tag{24}$$

where j^a and J^a are the dynamical equivalents of the electromagnetic current defined already. It is simple and obvious to see that the Cartan torsion form T^a is anti-symmetric – it is defined as a two-form, and a two-form is antisymmetric, having the required anti-symmetry properties of well-defined spin [1-12]. A spinning vector is illustrated in the appendices. The electrodynamic tetrad A^a_μ represents one frame spinning and propagating with respect to another. Hehl does not seem to understand this, even though he himself frequently admits that ECE theory is technically correct. He appears still to be thinking of a tetrad as an object in gravitational theory only, whereas in ECE theory it is more than that. Hehl speaks of “the trace of the first field equation”. Nowhere in ECE theory is such a trace mentioned, needed, or used. (Also see Appendix 11, item f.)

In Section 4.4, Hehl appears to disagree once again and reasserts incorrectly that there is an “error” in Cartan geometry, one that the present author has repeated. The truth is that ECE theory is based on the rigorously correct structure equations and Bianchi identities of Cartan, taught in all major universities. These are summarized in the appendices. Hehl appears to sometimes agree with this, and then to obscurely disagree.

The relation between index reduced canonical energy momentum density T and scalar curvature R in ECE theory is a generalization of Einstein’s well known

$$R = -kT \quad , \tag{25}$$

where k is the Einstein constant. This R is defined in Appendix J of Ref. [1].

Hehl now directs attention to the construction of a Lagrangian, and we are told that the present author does not use covariant derivatives in a Lagrangian. Hehl quotes Eq. (116) of UFT Paper 57 (available on aias.us). This was used to derive the ECE Lemma, Eq. (46) of the same paper, a Lemma which had already been derived from

$$D^{\mu}\left(D_{\mu} q_{\nu}^{a}\right) := 0 \quad , \tag{26}$$

i.e., from covariant derivatives.

Therefore, the Lagrangian has no need for covariant derivatives to derive an equation (46) in which there are ordinary derivatives already derived from covariant derivatives.

[5] Assessment

Hehl finally admits, in Section 5.1, that “the equations of Evans and associates (sic) are not very transparent to us”. Quite so, it is clear from the discourse that Hehl does not understand ECE theory (so why write a critique?).

Hehl then criticizes the convention on the normalization of tetrads. Due to orthogonality of the vielbeins we have

$$q_{a}^{\mu} q_{\nu}^{a} = \delta_{\nu}^{\mu} \tag{27}$$

and

$$q_{\mu}^{a} q_{b}^{\mu} = \delta_{b}^{a} \quad . \tag{28}$$

From this follows for the double sum:

$$q^a_\mu q^\mu_a = 4 \,. \tag{29}$$

The convention used in ECE theory is to normalize this to unity:

$$q^a_\mu q^\mu_a = 1 \,. \tag{30}$$

In this way, having to carry around factors of 4, and of ¼, in complicated calculations is avoided. The Einstein convention used in ECE theory is

$$g^{\mu\nu} g_{\mu\nu} = 4 \tag{31}$$

for the metric.

It is then asserted that equations (19), discovered by the present author in 2006 [1-12] were actually discovered in 1961. Hehl is confused because he does not understand that, while there may be occasional mathematical similarities to other theories, the ECE equations were derived within a different physics context, and thus have a completely different basis, interpretation and applicability.

ECE theory has nothing to do with the equations of Kibble or those of Sciama. Those are equations of general relativity including mechanical spin, while ECE theory includes electromagnetism and all forces of nature. If ECE theory were Kibble or Sciama, Hehl would not be criticizing it because, to this author's knowledge, he does not criticize Kibble or Sciama. It is this kind of obfuscation that renders this long Hehl document of little value to a discussion of ECE theory.

Hehl then asks what the present author means by spin. What Evans means by spin is given in references [1] to [12].

Hehl cites unrefereed and long rebutted material by Bruhn that claims that the wedge product of two tetrads cannot be made. The wedge product of two one-forms is a two-form [1-13] with the properties of anti-symmetry. This is a definition of ECE theory. Hehl chooses to deny a definition; nothing could be more subjective.

Towards the end of Section 5.2, there is some compounded obfuscation concerning the ECE wave equation. The latter is correctly derived in Appendix J of Ref. [1]. We are told in Section (5.3.1) that electrodynamics has nothing to do with the geometry of space-time. This is asserted without any reference at all to experimental data, for example, see *Advantages of ECE theory over the Standard Model* (aias.us).

ECE theory reduces, in appropriate limits, to all of the well-known laws of physics (thereby explaining all of the well-known experiments of physics consistently, within one framework, for the first time). It is to be noted that Francis Bacon advocated the testing of a theory with experimental data, not another theory or construct of the human mind. This is the long-accepted way of chemistry and physics. *If electrodynamics had nothing to do with geometry of space-time, then all of well-tested and very precise* *relativity theory would collapse, because relativity is based on geometry and must apply to all of physics, and not just gravitation.*

Sometimes Hehl states that Cartan is correct, sometimes he states that he is incorrect. What ECE theory means by Cartan geometry is standard and well defined, and reiterated in the mathematical appendices to this paper. The equivalence principle is well defined in ECE theory [1-12] because all particles have mass, including the photon. If Hehl means the equivalence of inertial mass and gravitational mass, then apply it to the photon and its mass to find that the equivalence principle exists in electrodynamics too. A paper on neutrino oscillations is actually available in Ref. [1] (also available as UFT Paper 20 on aias.us). Hehl seems to have overlooked this neutrino mass as well?

The ECE hypothesis:

$$A_\mu^a = A^{(0)} q_\mu^a \tag{32}$$

defines the electromagnetic potential as a vector valued one-form. The electromagnetic potential is Feynman's "universal influence", and the minimum is a universal constant (see the appendices). The universality of ECE theory enters through the usual Einstein constant, i.e., in the universal proportionality of $-R$ to T for ALL radiated and matter fields. The $cA^{(0)}$ is a primordial voltage [1-12] with minimum value $\frac{mc^2}{e}$, where mc^2 is the photon rest energy and e is the charge on the proton.

A matter field with spin, such as the electron or neutrino, is described in ECE theory by a wave-function that is a C positive tetrad. Similarly, any matter field without spin is described by a curvature tetrad. Thus, the spin of the electron is described in the special relativistic limit of ECE theory [1-12] by a C positive spinor with four components, the Dirac four-spinor. The latter does not contain $-e$. In order to describe the interaction of the Dirac electron with the classical electromagnetic field, the minimal prescription is used within the Dirac equation. The charge $-e$ enters into the description in this way, but the spin of the electron is described by a C positive spinor. In a particle such as the neutrino with spin and no charge, there is no $-e$ present.

The eigenfunction of the ECE wave equation is, therefore, the appropriate q_μ^a and not $A^{(0)} q_\mu^a$. The eigenfunction of the spin half electron is also the appropriate q_μ^a, but the eigenfunction of the spin one electromagnetic field is $A^{(0)} q_\mu^a$. The photon with mass is described by the Proca equation, with this eigenfunction.

All elementary particles and all four fundamental fields can thus be described by one equation – the ECE wave equation. This is the essence of ECE theory – all of known physics is now described within the same mathematical framework. Physics is geometry – objective and deterministic (as many scientists like Einstein always believed it had to be).

In general, therefore, all of the matter fields are described by

$$(\Box + kT)q_\mu^a = 0 \, , \tag{33}$$

and the various radiated fields by equations similar to

$$(\Box + kT)A_\mu^a = 0 \, . \tag{34}$$

In this respect, Hehl becomes confused and states incorrectly that the basic ECE hypothesis cannot be true. This appears to be subjective zeal, which is neither the scientific method nor the scientific spirit.

We are told that there is somehow no "charge - current conservation" in ECE theory. The argument by Hehl is so obscure that the following is the present author's interpretation of what he may mean. Hehl again argues in the context of what he incorrectly describes as Maxwell's theory. In fact, he describes Heaviside's theory, in which charge current conservation is described by

$$\frac{\partial J^\mu}{\partial x^\mu} = 0 \, , \tag{35}$$

where J is the inhomogeneous current.

In ECE theory, the equivalent equation is

$$\frac{\partial J^{\mu a}}{\partial x^\mu} = 0 \, , \tag{36}$$

and is true for each polarization described by the index a (transverse or longitudinal, and also time-like).

Charge current conservation is developed in several places in the three volumes [1-3], e.g., pp. 484, 508 and 515 of Volume One. Eq. (E.13) on page 484 is an example. This has the same structure as the Maxwell Heaviside equation for each index a. Here $a =$ (1) and (2) indicate complex conjugate transverse polarizations, and $a =$ (3) indicates longitudinal polarization.

A plane wave, for example, has polarizations (1) and (2), and it is well known that a plane wave conserves charge/current density. So, it is easy to show that ECE theory reduces to Heaviside's theory for each a and also conserves charge/current density for each a, i.e., for two senses of transverse circular polarization ($a =$ (1) and (2)), and one sense of longitudinal polarization ($a =$ (3)). The time-like sense is $a =$ (0). ECE theory thus reduces in a well-defined way [1-12] to a type of Heaviside theory in which the senses of polarization are well defined by the upper index $a =$ (1), (2), (3) of the potential A^a. The lower index denotes X, Y and Z in three-dimensional space.

Hehl then cites Bruhn's erroneous comment on the B Cyclic Theorem without citing the definitive rebuttal, which is given again in the appendices to this paper. He also cites a decade old paper by Lakhtakia, but again does not cite the decade old rebuttal of Lakhtakia's nonsensical claim that the ECE spin field is somehow not observable. It is observable in the inverse Faraday effect [1-12] and in several other ways to be found in the books and papers published by this author.

The energy momentum in ECE theory is defined for all particles and fields by

$$R = -kT \ , \tag{37}$$

and this is a universal proportionality because the constant k is universal and the same for all particles and fields. All particles and fields in ECE theory are unified by Cartan geometry. Hehl's comments on energy momentum are wholly irrelevant to this simple and clear definition. A subjective Lagrangian construction is mentioned by Hehl towards the end of his first document [14]. This then develops into a second document with another author of his own group.

Second Paper (Hehl and Obukhov)

In the approach to the second paper [15], it is mentioned that a Lagrangian multiplier has been found that eliminates torsion. This obscurity presumably means that the Lagrangian that gives zero torsion must be a Lagrangian of the Einstein Hilbert theory, which has no torsion. In what way can this argument "refute" ECE theory? The latter contains torsion by definition.

It is then stated in the abstract of the Hehl and Obukhov paper that a Lagrangian has been found that correctly reproduces the ECE field equations. We are apparently in agreement again. We are told in Eqs. (8) and (9) of that paper that the ECE equations are correct, despite their first paper. Not only are the ECE equations correct, but they can be derived by a Lagrangian. This omits to mention that all of this has already been done [1-12].

In Eq. (10) of the second paper, a Lagrangian is set up for what is again erroneously called Maxwell's theory – in actual fact it is Heaviside's theory. These Lagrangian methods are, however, already widely discussed in ECE theory [1-12]. In Eq. (13) another irrelevant Lagrangian appears. We are told that the electromagnetic field is massless, whereas in ECE theory the photon must have mass (highlighting another failure of the Standard Model). Therefore, the criticism in the second paper [15] is entirely irrelevant.

We are then told in Section 2.3 that although A^a_μ carries spin, as is evident in ECE theory, we are not actually told this by Evans. The potentials $A^{(1)}_X$ and so on from A^a_μ are described many times in ECE theory and precursor gauge theory as carrying spin, for example, they are described many times as propagating plane waves [1-12] that spin right or spin left as they propagate.

It is asserted incorrectly that A^a_μ transforms as a vector under Lorentz transformations. The correct generally covariant transform is

$$A^{a'}_{\mu'} = \Lambda^{a'}_a \frac{\partial x^\mu}{\partial x^{\mu'}} A^a_\mu \ , \tag{38}$$

and this is not a Lorentz transformation.

Again, A^a_μ is not a vector, it is a rank two mixed index tensor [13].

In Eq. (25) (of [15]) we finally arrive, after much verbiage, at a Lagrangian multiplier. We are told that this is necessary, but we are not told why. Shortly thereafter, the Lagrangian multiplier is discarded and the correct ECE field equations recovered. We are in agreement again, so why introduce a multiplier? The fact is that this is an entirely irrelevant procedure. When the multiplier is reinstated, the tautological deduction is made that the ECE field equations are changed. This is an entirely meaningless procedure. Any Lagrangian multiplier can be chosen, and this one was chosen to eliminate torsion from Cartan geometry. This procedure is incorrect because torsion is intrinsic to Cartan geometry (see the appendices).

In Section 4 of the second paper, it is incorrectly asserted that the second Cartan structure equation is not the second Cartan structure equation known to textbooks since 1922. The truth is that the second Cartan structure equation defines the Cartan curvature form:

$$R^a_b = d \wedge \omega^a_b + \omega^a_c \wedge \omega^c_b \quad . \tag{39}$$

It has been shown in all detail [1-12] that this correctly defines the standard Riemann tensor for any connection. This is necessary and sufficient.

We arrive, finally, at a conclusion**:** we are told that the torsion has been removed by a Lagrangian multiplier that makes the correct ECE equations incorrect. This is pure nonsense for the large list of reasons discussed in this rebuttal (and in more mathematical detail in the attached appendices, in published books and papers, and on the AIAS website). □

Appendix 1: The Standard Cartan Structure Equations and Identities Used in ECE Theory

The following are standard equations of differential geometry [13].

The first Cartan structure equation is

$$T^a = D \wedge q^a = d \wedge q^a + \omega^a_b \wedge q^b \ , \tag{A.1}$$

where T^a is the Cartan torsion form, q^a is the Cartan tetrad form, and ω^a_b is the spin connection.

After applying the standard tetrad postulate [13]:

$$D_\mu \wedge q^a_\nu = 0 \ , \tag{A.2}$$

Eq. (A.1) becomes the definition of the torsion tensor:

$$T^\kappa_{\mu\nu} = \Gamma^\kappa_{\mu\nu} - \Gamma^\kappa_{\nu\mu} \ , \tag{A.3}$$

where $\Gamma^\kappa_{\mu\nu}$ is the general connection of Riemann geometry.

Note that for the Christoffel connection:

$$\Gamma^\kappa_{\mu\nu} = 0 \ , \tag{A.4}$$

there is no torsion. This is the case in Einstein Hilbert theory.

The second structure equation of Cartan is

$$R^a = D \wedge \omega^a = d \wedge \omega^a + \omega^a_c \wedge \omega^c_b \ , \tag{A.5}$$

and applying the tetrad postulate (A.2) makes it equivalent to the definition of the Riemann tensor for any connection:

$$R^\sigma_{\lambda\nu\mu} = \partial_\nu \Gamma^\sigma_{\mu\lambda} - \partial_\mu \Gamma^\sigma_{\nu\lambda} + \Gamma^\sigma_{\nu\rho}\Gamma^\rho_{\mu\lambda} - \Gamma^\sigma_{\mu\rho}\Gamma^\rho_{\nu\lambda} \ . \tag{A.6}$$

Note that the Riemann tensor used in Einstein Hilbert theory is defined by the Christoffel connection.

The first identity of Cartan geometry is

$$D \wedge T^a := R^a_b \wedge q^b \ , \tag{A.7}$$

and applying the tetrad postulate (A.2) makes it equivalent to the cyclic sum [1-12]:

$$\begin{aligned} R^\lambda_{\rho\mu\nu} + R^\lambda_{\mu\nu\rho} + R^\lambda_{\nu\rho\mu} \ := \ & \partial_\mu \Gamma^\lambda_{\nu\rho} - \partial_\nu \Gamma^\lambda_{\mu\rho} + \Gamma^\lambda_{\mu\sigma}\Gamma^\sigma_{\nu\rho} - \Gamma^\lambda_{\nu\sigma}\Gamma^\sigma_{\mu\rho} \\ & + \partial_\nu \Gamma^\lambda_{\rho\mu} - \partial_\rho \Gamma^\lambda_{\nu\mu} + \Gamma^\lambda_{\nu\sigma}\Gamma^\sigma_{\rho\mu} - \Gamma^\lambda_{\rho\sigma}\Gamma^\sigma_{\nu\mu} \\ & + \partial_\rho \Gamma^\lambda_{\mu\nu} - \partial_\mu \Gamma^\lambda_{\rho\nu} + \Gamma^\lambda_{\rho\sigma}\Gamma^\sigma_{\mu\nu} - \Gamma^\lambda_{\mu\sigma}\Gamma^\sigma_{\rho\mu} \end{aligned} \tag{A.8}$$

for any connection.

Note that in Einstein Hilbert theory there is no torsion, so this identity reduces to

$$R^{\lambda}_{\rho\mu\nu} + R^{\lambda}_{\mu\nu\rho} + R^{\lambda}_{\nu\rho\mu} = 0 \ , \tag{A.9}$$

which becomes equivalent to

$$R^{a}_{b} \wedge q^{b} = 0 \ , \tag{A.10}$$

after applying the tetrad postulate (A.2).

Finally, the second identity of Cartan geometry is a restatement of the second structure equation:

$$D \wedge R^{a}_{b} := D \wedge (D \wedge \omega^{a}_{b}) \ . \tag{A.11}$$

Note that for a Christoffel connection, i.e., if and only if the torsion vanishes, Eq. (A.11) becomes

$$D \wedge R^{a}_{b} = 0 \tag{A.12}$$

and, after applying the tetrad postulate (A.2), it becomes equivalent to

$$D_{\rho} R^{\kappa}_{\sigma\mu\nu} + D_{\mu} R^{\kappa}_{\sigma\nu\rho} + D_{\nu} R^{\kappa}_{\sigma\rho\mu} = 0 \ . \tag{A.13}$$

The tetrad postulate [13] is true for any connection, and is the statement that a complete vector field must be independent of its components and basis elements. This is always true in any application in physics. In ECE theory the tetrad postulate is proven in many ways [1-12].

This defines the geometry of ECE theory, which is standard Cartan geometry. The detailed proofs of these statements are given in Appendices A, B, and C from Chapter 17 of Ref. [1], and in UFT Paper 88 (which supersedes Appendix D of Chapter 17 for the derivation of the second Bianchi identity of Cartan geometry).

If Hehl uses a different geometry, he is describing neither standard Cartan geometry nor ECE theory. □

Appendix 2: Proof of the Lorentz Invariance of the B Cyclic Theorem

One of the most serious errors by Hehl is his uncritical citation of a claim by Bruhn concerning the B Cyclic Theorem [1-12]. This alone brings into serious doubt Hehl's impartiality.

The B Cyclic Theorem is stated in the following way.

Consider the magnetic flux densities defined by the polarizations (1), (2) and (3):

$$\boldsymbol{B}^{(1)} = \frac{B^{(0)}}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j})e^{i\phi} \quad , \tag{B.1}$$

$$\boldsymbol{B}^{(2)} = \frac{B^{(0)}}{\sqrt{2}}(\boldsymbol{i} + i\boldsymbol{j})e^{-i\phi} \quad , \tag{B.2}$$

$$\boldsymbol{B}^{(3)} = B^{(0)}\mathbf{k} \quad . \tag{B.3}$$

Then

$$\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = iB^{(0)}\boldsymbol{B}^{(3)^*} \quad \text{et cyclicum} \tag{B.4}$$

is Lorentz invariant. Polarizations (1) and (2) are transverse complex conjugates, and ϕ is the electromagnetic phase. Polarization (3) is longitudinal and $\boldsymbol{B}^{(3)}$ is an observable of the inverse Faraday effect [1-12].

The proof of the Lorentz invariance is simple. By direct substitution it is found that Eq. (B.4) is

$$\mathbf{e}^{(1)} \times \mathbf{e}^{(2)} = i\mathbf{e}^{(3)^*} \quad \text{et cyclicum,} \tag{B.5}$$

where the unit vectors of the complex circular basis [1-12] are

$$\boldsymbol{e}^{(1)} = \frac{1}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j}) \quad ,$$

$$\boldsymbol{e}^{(2)} = \frac{1}{\sqrt{2}}(\boldsymbol{i} + i\boldsymbol{j}) \quad , \tag{B.6}$$

$$\boldsymbol{e}^{(3)} = \mathbf{k} \quad .$$

Under a Lorentz transformation the unit vectors i, j and k do not change, and so the B Cyclic Theorem is Lorentz invariant. Q.E.D.

Bruhn falsely claims that this is not so. The Lorentz transformation of a magnetic field is found in any textbook to be

$$\boldsymbol{B}' = \gamma\left(\boldsymbol{B} - \frac{1}{c^2} v \times \boldsymbol{E}\right) \ , \tag{B.7}$$

where v is a velocity, and where

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \ . \tag{B.8}$$

The Lorentz transform of an electric field is

$$\boldsymbol{E}' = \gamma(\boldsymbol{E} + v \times \boldsymbol{B}) \ . \tag{B.9}$$

It is seen that i, j, and k do not change, they are the same for $\boldsymbol{B}'$ and $\boldsymbol{B}$ and for $\boldsymbol{E}'$ and $\boldsymbol{E}$. The $\boldsymbol{B}^{(0)}$ factor changes, but is cancelled out on both sides of Eq. (B.4) to give Eq. (B.5). This was first shown nearly a decade ago [1-12], but was ignored by both Bruhn and Hehl. □

Appendix 3: Nature of the Constant $cA^{(0)}$

The least value of $cA^{(0)}$ may be worked out using the fundamental relations:

$$B^{(0)} = \kappa A^{(0)} \ , \tag{C.1}$$

where κ is the wavenumber:

$$\kappa = \frac{\omega}{c} \ ; \tag{C.2}$$

and

$$eA^{(0)} = \hbar\kappa \ , \tag{C.3}$$

where e is the proton charge, and $\hbar$ is the reduced Planck constant.

Using the de Broglie photon mass equation:

$$\hbar\kappa = mc^2 \ , \tag{C.4}$$

where m is the mass of the photon, it is concluded that the least value of the voltage $cA^{(0)}$ is

$$cA^{(0)} = \frac{mc^2}{e} \ . \tag{C.5}$$

This is a universal constant because the photon mass is a universal constant. The photon mass is essential for an understanding of light deflection by gravitation, as is well known. □

Appendix 4: Derivation of the Lorentz Force Equation in ECE Theory

This derivation has been available for some time [1-12], but Hehl appears to assert that it is not available. His obscurity is such that his meaning is not clear to the present author, so the derivation is repeated in this appendix.

The Lorentz force equation in ECE theory becomes an equation of general relativity and is derived from the transformation properties of the Cartan torsion [13]:

$$T^{a'}_{\mu'\nu'} = \Lambda^{a'}_{a} \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\nu}}{\partial x^{\nu'}} T^{a}_{\mu\nu} \ , \tag{D.1}$$

where $\Lambda^{a'}_{a}$ denotes Lorentz transformation and $\frac{\partial x^{\mu}}{\partial x^{\mu'}}$ denotes coordinate transformation.

In the special case where rotation and translation are mutually independent, the ECE field equations are

$$d \wedge F^{a}_{\mu\nu} = 0 \ , \tag{D.2}$$

$$d \wedge \tilde{F}^{a}_{\mu\nu} = \mu_0 J^{a}_{\rho\mu\nu} \ , \tag{D.3}$$

in tensor notation [1-12].

For each polarization index a, these have the same structure as the Maxwell Heaviside field equations:

$$d \wedge F_{\mu\nu} = 0 \ , \tag{D.4}$$

$$d \wedge \tilde{F}_{\mu\nu} = \mu_0 J_{\rho\mu\nu} \ . \tag{D.5}$$

It is well known that $F_{\mu\nu}$ in Eq. (D.4) transforms as

$$F_{\mu'\nu'} = \Lambda^{\mu}_{\mu'} \Lambda^{\nu}_{\nu'} F_{\mu\nu} \tag{D.6}$$

for each state of polarization a.

Eq. (D.6) is the limit:

$$\frac{\partial x^{\mu}}{\partial x^{\mu'}} \rightarrow \Lambda^{\mu}_{\mu'} \quad ; \quad \frac{\partial x^{\nu}}{\partial x^{\nu'}} \rightarrow \Lambda^{\nu}_{\nu'} \quad ; \tag{D.7}$$

for each fixed a.

The Lorentz force equation is obtained from Eq. (D.6) as

$$\boldsymbol{E}' = \gamma(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) + \cdots \ , \tag{D.8}$$

$$\boldsymbol{B}' = \gamma\left(\boldsymbol{B} - \frac{1}{c^2}\boldsymbol{v} \times \boldsymbol{E}\right) + \cdots \ , \tag{D.9}$$

and

$$\boldsymbol{F}' = e\gamma(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) + \cdots \ . \tag{D.10}$$

Therefore, for each sense of polarization:

$$\boldsymbol{F}^{(1)'} = e\gamma\left(\boldsymbol{E}^{(1)} + \boldsymbol{v} \times \boldsymbol{B}^{(1)}\right) \ , \tag{D.11}$$

$$\boldsymbol{F}^{(2)'} = e\gamma\left(\boldsymbol{E}^{(2)} + \boldsymbol{v} \times \boldsymbol{B}^{(2)}\right) \ , \tag{D.12}$$

$$\boldsymbol{F}^{(3)'} = e\gamma\left(\boldsymbol{E}^{(3)} + \boldsymbol{v} \times \boldsymbol{B}^{(3)}\right) \ . \tag{D.13}$$

The third equation reduces to [1-12]:

$$\boldsymbol{F}^{(3)'} = -ieg\gamma v \times \left(\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}\right) \ , \tag{D.14}$$

because there is no electric equivalent of the inverse Faraday effect:

$$\boldsymbol{E}^{(3)} = 0 \ , \tag{D.15}$$

and because

$$\boldsymbol{B}^{(3)} = -ig\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)} \ , \tag{D.16}$$

where

$$g = \frac{e}{\hbar} = \frac{\kappa}{A^{(0)}} \ . \tag{D.17}$$

The object $A^{(1)} \times A^{(2)}$ is the well-known conjugate product of non-linear optics [1-12]. Due to the Lorentz invariance of the B Cyclic Equation, the polarizations:

$$a = (1), (2), (3) \tag{D.18}$$

are also Lorentz invariant, contrary to an obscure assertion by Hehl.

In the most general case, the Lorentz force equation in ECE theory must be defined by equation (D.1) and the ECE hypothesis accepted by Hehl:

$$F^{a}_{\mu\nu} = A^{(0)} T^{a}_{\mu\nu} \ . \tag{D.19}$$

□

Appendix 5: Spinning Space-Time

Hehl exhibits a complete lack of understanding of the nature of the electrodynamic tetrad introduced in ECE theory and defined by

$$A_\mu^a = A^{(0)} q_\mu^a \ , \tag{E.1}$$

and he appears to assert that this is a curvature tetrad of gravitational theory. The electrodynamic tetrad is defined by one frame spinning and propagating with respect to another. The frame is labeled *a* and μ is the coordinate index, and they are related to each other by the tetrad:

$$\boldsymbol{q}^{(1)} = \frac{1}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j})e^{i(\omega t - \kappa z)} \ . \tag{E.2}$$

This is a new application of the basic definition of a tetrad by Cartan geometry. In the gravitational application, the a index represents an orthonormal space (*a*) tangent to the base manifold (μ) at point P. In the electrodynamic application, frame a spins with respect to frame μ. For a plane wave, the tetrads [1-12] (in vector notation) are as follows:

$$\boldsymbol{q}^{(1)} = \frac{1}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j})e^{i\phi} \ , \tag{E.3}$$

$$\boldsymbol{q}^{(2)} = \frac{1}{\sqrt{2}}(\boldsymbol{i} + i\boldsymbol{j})e^{-i\phi} \ , \tag{E.4}$$

$$\boldsymbol{q}^{(3)} = \boldsymbol{k} \ , \tag{E.5}$$

where

$$\phi = \omega t - \kappa Z \tag{E.6}$$

is the electromagnetic phase. Here ω is the angular frequency of spin at instant t, and κ is the wave-number of propagation at point Z. Therefore, the plane wave is one of a spinning and propagating frame. The spin connection is needed to define this frame.

Hehl confuses this with the curving of space-time in gravitational theory, a basic error that renders his whole document irrelevant.

The spinning and propagating tetrad in three dimensions, for example, is defined by [1-12]:

$$V^a = \begin{bmatrix} q_X^{(1)} & q_Y^{(1)} & q_Z^{(1)} \\ q_X^{(2)} & q_Y^{(2)} & q_Z^{(2)} \\ q_X^{(3)} & q_Y^{(3)} & q_Z^{(3)} \end{bmatrix} V_\mu \ . \tag{E.7}$$

The column vector V^a is

$$V^a = \begin{bmatrix} \frac{1}{\sqrt{2}}(1-i)e^{i\phi} \\ \frac{1}{\sqrt{2}}(1+i)e^{-i\phi} \\ 1 \end{bmatrix} , \tag{E.8}$$

and the column vector V^μ is

$$V^\mu = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} . \tag{E.9}$$

The column vector V^a rotates (i.e., spins) as follows:

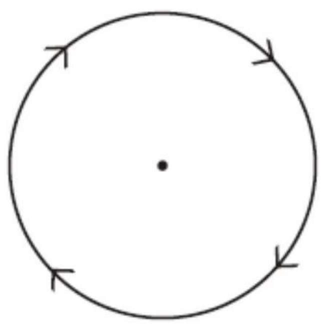
$$V^a = \frac{1}{\sqrt{2}} \begin{bmatrix} \cos\ \phi + \sin\ \phi \\ \cos\ \phi - \sin\ \phi \\ \sqrt{2} \end{bmatrix} \tag{E.10}$$

$$\phi = 0, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \nearrow$$

$$\phi = \frac{\pi}{2}, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \searrow$$

$$\phi = \pi, \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \swarrow \tag{E.11}$$

$$\phi = \frac{3\pi}{2}, \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \nwarrow$$



and the μ frame is fixed. The spin in ECE theory is clearly defined in this and many other ways [1-12]. The only counter argument is to assert that one frame cannot spin with respect to another, which is a reduction to absurdity. Finally, the interaction of fields is defined by combined curving and spinning. □

Appendix 6: Criticisms of the Gauge Principle

There are many criticisms available [1-12] of the gauge principle, which is used extensively in Hehl's work, and this is a backward step in general relativity. In ECE theory, the problems with the gauge principle are removed by using the invariance of the tetrad postulate under general coordinate transformation. The gauge principle is based on a late nineteenth century assumption that the electromagnetic potential is a mathematical convenience. This contradicted the views of Faraday and Maxwell, and has always been in dispute, with an example being the fifty-year debate over the Aharonov Bohm effect. Such a protracted debate means that the question is open.

Another well-known example of the failure of U(1) gauge invariance in electrodynamics is the Proca equation:

$$\left(\square + \frac{m^2c^2}{\hbar^2}\right) A^{\mu} = 0 \ , \tag{F.1}$$

where m is the photon mass and where A^{μ} is the U(1) potential of the Standard Model. The Lagrangian methods of the Standard Model run into insurmountable difficulties when addressing the Proca equation, which is not U(1) gauge invariant. In ECE theory, the Proca equation is obtained from the tetrad postulate and the hypothesis:

$$A^a_{\mu} = A^{(0)} q^a_{\mu} \ , \tag{F.2}$$

giving the generally covariant wave equation of electrodynamics:

$$(\square + kT) A^a_{\mu} = 0 \ . \tag{F.3}$$

In the limit where the electromagnetic field is free of the influence of other fields (such as gravitation), Eq. (F.3) reduces to

$$\left(\square + \frac{m^2c^2}{\hbar^2}\right) A^a_{\mu} = 0 \ , \tag{F.4}$$

which is covariant, as required. There is no internal conflict as in the Standard Model.

The equation:

$$kT = k\frac{m}{V} = \frac{m^2c^2}{\hbar^2} \tag{F.5}$$

defines the finite volume:

$$V = \frac{k\hbar^2}{mc^2} \ . \tag{F.6}$$

In general, this equation means that there are no singularities in nature – every particle occupies a finite volume, and thus the need for renormalization is removed. Therefore, Hehl's attempt to use gauge theory in general relativity is bound to fail, because it is based on a flawed concept in special relativity (the concept that the potential in Maxwell Heaviside theory is unphysical). To adopt a flawed concept for general relativity has no purpose, and to attempt to criticize ECE theory with a flawed concept is doubly inappropriate. The failure of U(1) gauge invariance to produce the Proca equation, an equation that is fundamental to physics, is fatal for gauge theory. The photon mass is observable to a precision of one part in a hundred thousand in the Solar System by measuring the deflection of light due to gravitation. Without photon mass, there would be no such deflection. □

Appendix 7: Refutation of U(1) Gauge Invariance in the Inverse Faraday Effect

The inverse Faraday effect is the magnetization of matter by a circularly polarized electromagnetic field at any frequency. The magnetization is

$$\boldsymbol{M}^{(3)} = -ig'\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)} \ , \tag{G.1}$$

where g' is a material property, and for a plane wave:

$$\boldsymbol{A}^{(1)} = \frac{\boldsymbol{A}^{(0)}}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j})e^{i\phi} \ , \tag{G.2}$$

$$\boldsymbol{A}^{(2)} = \frac{\boldsymbol{A}^{(0)}}{\sqrt{2}}(\boldsymbol{i} + i\boldsymbol{j})e^{-i\phi} \ . \tag{G.3}$$

Under U(1) gauge transformation:

$$A^{(1)} \rightarrow A^{(1)} + \nabla x^{(1)} \ ,$$

$$A^{(2)} \rightarrow A^{(2)} + \nabla x^{(2)} \ , \tag{G.4}$$

so the inverse Faraday effect is changed.

This is a counter-example to U(1) gauge invariance in the Standard Model, because the gauge principle asserts that a physical observable CANNOT be changed by a U(1) gauge transformation. This rule happens to be true for the electric field and the magnetic field, where changing the potential under a U(1) gauge transform has no effect, but it is no longer true in the inverse Faraday effect, which is an example of a non-linear optical effect. This is one counter-example out of many to Hehl's arbitrary assertion that the electromagnetic potential is unphysical.

In ECE theory the inverse Faraday effect is due to the spin connection of spinning space-time, and is a special case of the second term on the right-hand side of

$$F^a = d \wedge A^a + \omega^a_b A^b \ . \tag{G.5}$$

When the spin connection is dual to the tetrad, the $\boldsymbol{B}^{(3)}$ spin field [1-12] is defined:

$$\boldsymbol{B}^{(3)} = -ig\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)} \ , \tag{G.6}$$

and the magnetization of the inverse Faraday effect is

$$\boldsymbol{M}^{(3)} = \frac{1}{\mu}\frac{g'}{g}\boldsymbol{B}^{(3)} \ . \tag{G.7}$$

When the spin connection is dual to the tetrad, the electromagnetic field becomes independent of the gravitational field in free space, and the spinning frame that defines the electromagnetic field becomes independent of the curving frame that defines the gravitational field. This condition is defined by the fact that the rotational part of the Riemann form is dual to the torsion form:

$$R^a_b = \frac{-\kappa}{2}\epsilon^a_{bc}T^c \quad . \tag{G.8}$$

This condition is further discussed in Appendix 8, which summarizes the basic mathematical advances made in ECE theory. □

Appendix 8: Mathematical Advances in ECE Theory

Hehl has entirely failed to understand the basic mathematical advances made in ECE theory [1-12]. His argument is still based on the concept of curving space-time, in which both torsion and curvature are aspects of gravitational theory only. In that theory, there is an orthonormal and orthogonal [13] tangent space-time at point P to a base manifold.

It is shown in Appendix 5, that the tetrad may be used for spinning space-time in a well-defined mathematical manner. In the case of the spinning tetrad, one frame spins and simultaneously propagates with respect to another. The tetrad is defined by two column vectors as usual, but its interpretation is different from that in gravitational theory, and akin to that in gauge theory. However, the abstract indices of the fiber bundle of gauge theory are replaced by the geometrical understanding of Appendix 5, and as argued in Appendices 6 and 7. Consequently, the gauge principle has been abandoned in ECE theory. One of the mathematical advances made in ECE theory was to develop the Riemann form for any connection, so that it becomes defined for mathematically well-defined spinning space-time as well as curving space-time.

When the tetrad describes spin motion of space-time (Appendix 5) unaffected by curving of space-time, the homogeneous current of ECE theory vanishes:

$$j^a = \frac{A^{(0)}}{\mu_0}\left(R^a_b \wedge q^b - \omega^a_b \wedge T^b\right) = 0 \ . \quad \text{(H.1)}$$

It has been shown in Appendix J of Ref. [1] that this condition implies

$$\omega^a_b = \frac{-\kappa}{2} \in^a_{bc} q^c \ . \quad \text{(H.2)}$$

The spin connection becomes dual to the tetrad through a wave-number. Furthermore, the Riemann form becomes dual to the torsion form through the same wave-number:

$$R^a_b = \frac{-\kappa}{2} \epsilon^a_{bc} T^c \ . \quad \text{(H.3)}$$

This is a fundamental mathematical advance of ECE theory. The ECE duality equations define a hitherto unknown rotational part of the Riemann form:

$$R^a_{b\mu\nu} = q^a_b q^\kappa_b R^\rho_{\kappa\mu\nu} \ , \quad \text{(H.4)}$$

where $R^\rho_{\kappa\mu\nu}$ is the Riemann tensor.

In Einstein Hilbert (EH) theory the duality (H.3) is not present, because in EH theory the Riemann form is a pure curvature form. There is no torsion in EH theory, so the Riemann form of EH is non-zero and the torsion form of EH is zero. Therefore, one cannot be the dual of the other.

Hehl's use of torsion is restricted to gravitational theory, where the first Bianchi identity of Appendix 1 implies that the Ricci cyclic equation for curving space-time may be written in terms of torsion. This identity asserts that the Riemann tensor for any connection is identically equal to itself. In gravitational theory, there is no notion of spinning space-time as defined in Appendix 5.

The spinning space-time of Appendix 5 may, therefore, be described by a new type of Riemann form. This is not a curvature form but rather a tensor valued two-form describing spin. It is the dual tensor of the vector valued torsion two-form describing spin. The form indices (the Greek indices) are fixed, so this is an example of an antisymmetric tensor being dual to an axial vector, representing spin. In general, the Riemann form is made up of a combination of curvature and spin parts. Prior to ECE theory, this appears not to have been known in mathematics, or at least not clearly defined.

The definition of the spin tetrad still uses the concept of one orthogonal and orthonormal frame (the static frame of Appendix 5) and one dynamical frame. Each frame is defined by a column vector, so the definition of the tetrad is unchanged:

$$V^a = q^a_\mu V^\mu \quad . \tag{H.5}$$

The tetrad, as in gravitational theory, is a matrix that links the column vectors. Therefore, ECE theory is a unified field theory because the field sectors are all self-consistently defined by the tetrad, which is always the fundamental field. The interaction of fields is then defined by the Cartan structure equations and Bianchi identities of Appendix 1. Different mathematical representation spaces may be used for the column vectors in Eq. (H.5). For a fermion, they are defined using an SU(2) representation space. In strong field theory, an SU(3) representation space is used.

A major mathematical advance of ECE theory has been to derive the generally covariant Dirac equation from the tetrad postulate, and to recognize that the Dirac spinor can itself be represented by tetrad elements. The usual method used in the Standard Model is to make the Dirac matrix covariant, while still using the Minkowski space-time for the spinor. This is self-inconsistent.

The generally covariant Dirac equation of ECE theory is internally consistent. The fermion wave-function is the tetrad. Therefore, ECE theory has unified general relativity and quantum mechanics, and self-consistently reduces to all of the major equations of physics in appropriate limits. This is achieved within a causal, deterministic framework as advocated by Einstein and de Broglie.

Another major advance was to realize that the Cartan structure equations and Bianchi identities of Appendix 1 apply to spin as well as curvature. Once the tetrad is defined as in Appendix 5, the torsion is defined by the first structure equation. The major advance in physics of ECE theory was the realization that the spin tetrad is the potential field of electromagnetism, and that the Cartan spin torsion is the anti-symmetric electromagnetic field for each index a, representing states of polarization.

None of these advances seem to be understood by Hehl. □

Appendix 9: Short Summary of Trivial Errors by Bruhn

This appendix is a short summary of numerous trivial errors by G. Bruhn that illustrate the fact that Bruhn deliberately tries to obscure basic mathematical facts that are well known and have been used for years. Hehl cites these errors uncritically, but does not cite my rebuttals.

a) He asserts that

$$\mathbf{e}^{(3)} = \mathbf{k} \tag{I.1}$$

is "undefined". If so, the Cartesian coordinate system would be undefined, which is an absurd claim.

b) He asserts that the right-hand side of

$$g^{\mu\nu} g_{\mu\nu} = 4 \tag{I.2}$$

is not a scalar, again an absurd claim.

c) He incorrectly sums up left and right circular polarization to give

$$\boldsymbol{B} =?\ \frac{\boldsymbol{B}^{(0)}}{\sqrt{2}} (\boldsymbol{i} - i\boldsymbol{j}) \cos\ \phi\ , \tag{I.3}$$

which he claims wrongly to be linear polarization. The correct summation is

$$\boldsymbol{B}_L^{(1)} + \boldsymbol{B}_R^{(1)} = \frac{2}{\sqrt{2}} B \times \boldsymbol{i} e^{i\phi}\ . \tag{I.4}$$

d) He incorrectly asserts that the tetrad is a vector with a label on it, whereas the tetrad is well known to be a rank two mixed index tensor.

e) He asserts that the tetrad postulate is

$$D_\mu X_b^a =?\ 0\ , \tag{I.5}$$

whereas the correct tetrad postulate is

$$D_\mu q_\nu^a = 0\ . \tag{I.6}$$

f) He asserts that the tetrad postulate is incorrect, whereas it is true (Appendix 1) for all applications in physics.

□

Appendix 10: Rebuttal of G. Bruhn's Comments on the Lorentz Covariance of The B Cyclic Theorem

It is well known that special relativity demands that any equation of physics be covariant under the Lorentz transformation [13]. It has been shown elsewhere [8], that the B Cyclic Theorem is correctly Lorentz covariant. This is self-evident from the fact that the B Cyclic Theorem is, within Lorentz invariant phase factors, the frame of reference itself, and it is well known that a frame of reference is Lorentz covariant.

Bruhn [14] incorrectly asserts that a frame of reference must be Lorentz invariant. This is nonsense, because if it were true, the Lorentz transform would result in no change. It is well known that it results in the Fitzgerald Lorentz length contraction.

In this appendix, the correct Lorentz transformation of the B Cyclic Theorem is given. However, it was also given correctly in reference [8], which Bruhn cites incorrectly [14].

The B Cyclic Theorem is [8]:

$$\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = iB^{(0)}\boldsymbol{B}^{(3)^*}$$
$$\text{et cyclicum,} \qquad \text{(J.1)}$$

where the plane wave magnetic flux densities are

$$\boldsymbol{B}^{(1)} = \boldsymbol{B}^{(2)^*} = \frac{B^{(0)}}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j})e^{i\phi} \ ,$$
$$\boldsymbol{B}^{(3)} = \boldsymbol{B}^{(3)^*} = B^{(0)}\boldsymbol{k} \ . \qquad \text{(J.2)}$$

Here, the electromagnetic phase factor is

$$\exp\,(i\phi) = \exp\,(i(\omega t - \kappa Z)) \ , \qquad \text{(J.3)}$$

where omega is the angular frequency at instant t, and κ is the wave-number at position Z.

It is well known [13] that the phase factor is Lorentz invariant. The frame of reference being used is the complex circular basis [8]:

$$e^{(1)} \times e^{(2)} = ie^{(3)^*} \ , \qquad \text{(J.4)}$$

whose unit vectors are related to the Cartesian unit vectors by

$$\mathbf{e}^{(1)} = \mathbf{e}^{(2)^*} = \frac{1}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j}) \ ,$$
$$\mathbf{e}^{(3)} = \mathbf{e}^{(3)^*} = \mathbf{k} \ , \qquad \text{(J.5)}$$

where * denotes "complex conjugate".

The general rule [13] for the covariant transformation of a basis vector is well known, and is

$$\hat{e}_{(\mu')} = \left(\frac{\partial x^{\mu}}{\partial x^{\mu'}}\right) \hat{e}_{(\mu)} \ , \tag{J.6}$$

where

$$x^{\mu} = (ct, X, Y, Z) \ , \tag{J.7}$$

and where the prime denotes a different frame of reference moving arbitrarily with respect to the original (un-primed) frame of reference.

The rule (J.6) is found in innumerable textbooks and shows that basis elements such as unit vectors are changed under the transformation. In other words, they are COVARIANT.

The Lorentz transform is a special case:

$$\hat{\mathrm{e}}_{(a)'} = \Lambda^{a}_{a'}(x)\hat{\mathrm{e}}_{(a)} \ , \tag{J.8}$$

as given in Chapter 3 of Ref. [13], and in innumerable textbooks for over a hundred years.

The complete vector field [13]:

$$V = V^{\mu}\hat{e}_{(\mu)} \tag{J.9}$$

is invariant under the transformation:

$$V = V^{\mu}\hat{e}_{(\mu)} = V^{\mu'}\hat{e}_{(\mu')} \ . \tag{J.10}$$

Therefore, the rule for general coordinate transformation [13] is

$$V^{\mu'} = \left(\frac{\partial x^{\mu'}}{\partial x^{\mu}}\right) V^{\mu} \ . \tag{J.11}$$

The Lorentz boost considers a frame of reference moving at constant velocity in an axis with respect to another frame. If this is the Z axis, then

$$\boldsymbol{V} = V_Z\boldsymbol{k} = V_{Z'}\boldsymbol{k}' \ , \tag{J.12}$$

from Eq. (J.10).

In general, the Lorentz transformation produces the result:

$$\boldsymbol{i} \times \boldsymbol{j} = \boldsymbol{k}$$

$$\boldsymbol{i}' \times \boldsymbol{j}' = \boldsymbol{k}' \tag{J.13}$$

in the Cartesian basis, or

$$\mathbf{e}^{(1)} \times \mathbf{e}^{(2)} = i\mathbf{e}^{(3)^*}$$

$$\mathbf{e}^{(1)'} \times \mathbf{e}^{(2)'} = i\mathbf{e}^{(3)^{*'}} \tag{J.14}$$

in the complex circular basis. These results follow directly from Eq. (J.8).

In the B Cyclic Theorem (J.1), the following definitions are used:

$$\boldsymbol{B}^{(1)} = \boldsymbol{B}^{(2)^*} = B^{(0)} e^{i\phi} e^{(1)} \ ,$$

$$\boldsymbol{B}^{(3)^*} = \boldsymbol{B}^{(3)} = B^{(0)} \boldsymbol{k} \ . \tag{J.15}$$

It is well known [13] that the electromagnetic phase is Lorentz invariant, because ϕ is a scalar, therefore,

$$e^{i\phi} = e^{i\phi'} \ . \tag{J.16}$$

Consequently, we need only consider the Lorentz covariance of $B^{(0)}$ and the frame itself (the unit vectors). If the boost takes place in Z, the complete vector field to be considered is

$$\boldsymbol{B}^{(3)} \to \boldsymbol{B}^{(3)} \ . \tag{J.17}$$

This means that

$$B^{(0)} \boldsymbol{k} = B^{(0)'} \boldsymbol{k}' \ ,$$

$$B^{(0)} \boldsymbol{i} = B^{(0)} \boldsymbol{i} \ , \tag{J.18}$$

$$B^{(0)} \boldsymbol{j} = B^{(0)} \boldsymbol{j} \ .$$

It has been shown that, in this case,

$$\boldsymbol{B}^{(0)'} = \left(\frac{1-\frac{v}{c}}{1+\frac{v}{c}} \right)^{\frac{1}{2}} B^{(0)} \ . \tag{J.19}$$

Equation (J.19) follows from the Lorentz transformation, where v is the velocity of the primed frame with respect to the un-primed frame.

The Cartesian frame after Lorentz transformation is

$$\boldsymbol{i} \times \boldsymbol{j} = \boldsymbol{k}' \ , \tag{J.20}$$

and the complex circular frame after Lorentz transformation is

$$\mathbf{e}^{(1)} \times \mathbf{e}^{(2)} = i\boldsymbol{e}^{(3)'} \ . \tag{J.21}$$

The frames are distorted, indicating the length contraction, but the overall form of the cyclic relations (J.20) and (J.21) is the same as the original cyclic relations (J.1) and (J.4). This is what is meant by COVARIANCE.

Furthermore, as indicated in Ref. [8], the velocity v is zero, because the fields are already propagating at c, and cannot propagate any faster, therefore,

$$B^{(0)'} = B^{(0)} \quad . \tag{J.22}$$

The above is elementary and well known. Why should such a basic error by Bruhn be published? □

Appendix 11: Rebuttal of Additional Comments by Hehl

a) New Footnote about Spin and Torsion

We are told that torsion is the global spin of all matter. This conclusion is a figment of Dr. Hehl's imagination. The UFT papers show clearly that the spin of the electromagnetic and fermion fields, for example, derives from the definition of the tetrad as the matrix linking one frame rotating with respect to another. This point had already been made in an earlier rebuttal.

It was shown by Beth at Princeton in 1936 that the electromagnetic field has angular momentum, which in field theory is a property derived from spin generators of a group. So angular momentum is a space-time property, as is well known to all textbooks, but not to Dr. Hehl. In ECE theory, the electromagnetic field is proportional to the torsion form, and the electromagnetic potential is proportional to the tetrad form, as defined to describe one frame rotating with respect to another (see earlier appendices).

b) New Footnote on Page 10

Bruhn contrives "errors" where none actually exist. Appendix 10, for example, shows that neither Hehl nor Bruhn understand the meaning of the Lorentz transform, but still force basic errors into print.

c) Page 14 of the Revised Version

Here there are many basic errors condensed into two sentences.

Dr. Hehl mistakes the Maxwell quaternion theory for the Maxwell Heaviside (MH) vector theory. The MH equations in free space, in form notation, are well known to be

$$d \wedge F = 0 \ , \tag{K.1}$$

$$d \wedge \tilde{F} = 0 \ , \tag{K.2}$$

where $\tilde{F}$ is the Hodge dual of F, which are the scalar valued field two-forms.

Therefore, these equations are not independent as Hehl tries to assert. As every standard textbook will state, they are Lorentz covariant in a Minkowski space-time. They are not generally covariant, as Hehl asserts. The Lorentz transformation produces well-known physical properties of classical electrodynamics in the limit of special relativity. These equations do not use covariant derivatives, so are not generally covariant. The space-time is flat, so there is no connection. However, we are suddenly told the contrary by Dr. Hehl, which is a glaring example of a contrivance.

The field form of the MH theory is defined by

$$F = d \wedge A \ , \tag{K.3}$$

where A is the scalar valued potential one-form. The Poincare Lemma then states that

$$d \wedge (d \wedge A) := 0 \ , \tag{K.4}$$

and this result has nothing to do with magnetic flux, another contrivance by Dr. Hehl. The Poincare Lemma is a result of topology.

The inhomogeneous current of MH theory is derived from the data, and this is a weakness of the MH theory precisely because of the fact that it is not a geometrical theory. In their correctly covariant form (given by ECE theory), the homogeneous and inhomogeneous currents may be derived from each other, and are not independent.

d) Footnote on Page 15

This comment has no basis, because the calculations of Evans are correctly based on Cartan geometry as described by Carroll (see previous comments).

e) New Section 3.3

This repeats the same error as the old Section 3.3. The correct method of doing a coordinate transformation on

$$D \wedge F^a = R^a_b \wedge A^b \tag{K.5}$$

is as follows:

$$\begin{array}{c} D \wedge F^a = R^a_b \wedge A^b \\ \downarrow \\ (D \wedge F^a)' = \left(R^a_b \wedge A^b\right)' \ . \end{array} \tag{K.6}$$

The equation is correctly covariant because it is based directly on the correctly covariant Bianchi identity. This is entirely true irrespective of Dr. Hehl's contorted efforts to convince readers to think otherwise.

To reiterate, the correct way of carrying out the Hodge dual transform is as follows:

$$\begin{array}{c} D \wedge F^a = R^a_b \wedge A^b \\ \downarrow \\ \left(D \wedge \hat{F}^a\right) = \left(\hat{R}^a_b \wedge A^b\right) \ . \end{array} \tag{K.7}$$

f) New footnote on Page 24

As mentioned previously, ECE theory neither needs nor uses “the trace of the first field equation”. It seems that Dr. Hehl is using a different definition for the term “antisymmetric metric”; therefore, he is criticizing something other than ECE theory. A “trace operation” is not necessary under the definition that Evans has used, and the argument of “RC space-time” does not apply. Hehl seems to have identified the metric with the tetrad, which is incorrect. Furthermore, the generalized electromagnetic potential of ECE theory is not antisymmetric, but rather has no internal symmetry.

□

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(Specific items that were referenced can be found in the following appendices**:** E (with page 484), I (with page 508), J, and K (with page 515), and Appendices A, B and C from Chapter 17. The first four are from the book proper, and the last three are from the chapter within the book. For convenience, these appendices have been excerpted, and are linked from *A Discussion of Criticisms of ECE Theory*, which is linked from the homepage article on the AIAS website (www.aias.us).)

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[15] Hehl, F. W., and Obukhov, Y. N. (2007), "An Assessment of Evans' Unified Field Theory II", *Foundations of Physics* 38(1). □